Branch-and-Cut algorithm for the connected-cut problem

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R\'esum\'e

Let $G=(V,E)$ be an undirected connected graph. Let $W$ be a subset of $V$, distinct from $V$. The set $W$ is said proper when it is non-empty. We denote by $\delta(W)$, the set of edges of $E$ having exactly one endnode in $W$. A set $F$ is called a cut of $G$, if there exists a proper subset $W$ of $V$ such that $F=\delta(W)$. Note that removing from a graph the edges of a cut can leave more than 2 connected components. Consequently, a cut is said to be connected if both subgraphs $G[W]=(W,E(W))$ and $G[V]=(V,E(V))$ are connected.

Let $c$ be a cost function defined on the edges of $E$, the maximum cut problem (Max-C) consists in finding a cut of maximum weight (where the weight of a cut is given by the summation of the costs of all the edges composing the cut). The particular case where the cost function is non-positive is called the minimum cut problem (Min-C). In a similar way, we can define the maximum (resp. minimum) connected cut problem Max-CC (resp. Min-CC) as finding a maximum cost connected cut.

It is easily seen that an optimal solution of the Min-C will be a connected cut and thus an optimal solution of the Min-CC. Consequently the Min-CC can be solved in polynomial time. The Max-C is strongly NP-hard. However, when the graph is planar, the Max-C problem can be solved in polynomial time [1], whereas the Max-CC is still NP-hard [2].

In this talk, we present integer formulations for the Max-CC together with preliminary results on the associated polytope. We also propose a Branch-and-Cut algorithm in order to solve instances generated from the TSPLIB.


Mots-Clés: Graph, Branch, and, cut, polyedral approach