
Branch-and-Cut algorithm for the connected-cut problem

Sylvie Borne¹, Pierre Fouilhoux^{*2}, Roland Grappe¹, Mathieu Lacroix¹, and Pierre Pesneau³

¹Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS, (UMR 7030), F-93430, Villetaneuse, France. –
Université Paris XIII - Paris Nord – France

²Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6 (UPMC - LIP6) – Université Pierre et Marie Curie - Paris 6 – 4 place Jussieu, 75005 Paris, France

³Université de Bordeaux, , IMB UMR 5251, INRIA Bordeaux - Sud-Ouest, 351 Cours de la libération, 33405 Talence – Université Bordeaux – France

Résumé

Let $G=(V,E)$ be an undirected connected graph. Let W be a subset of V , distinct from V . The set W is said proper when it is non-empty. We denote by $\delta(W)$, the set of edges of E having exactly one endnode in W . A set F is called a cut of G , if there exists a proper subset W of V such that $F=\delta(W)$. Note that removing from a graph the edges of a cut can leave more than 2 connected components. Consequently, a cut is said to be connected if both subgraphs $G[W] = (W, E(W))$ and $G[V] = (V, E(V))$ are connected.

Let c be a cost function defined on the edges of E , the maximum cut problem (Max-C) consists in finding a cut of maximum weight (where the weight of a cut is given by the summation of the costs of all the edges composing the cut). The particular case where the cost function is non-positive is called the minimum cut problem (Min-C). In a similar way, we can define the maximum (resp. minimum) connected cut problem Max-CC (resp. Min-CC) as finding a maximum cost connected cut.

It is easily seen that an optimal solution of the Min-C will be a connected cut and thus an optimal solution of the Min-CC. Consequently the Min-CC can be solved in polynomial time. The Max-C is strongly NP-hard. However, when the graph is planar, the Max-C problem can be solved in polynomial time [1], whereas the Max-CC is still NP-hard [2].

In this talk, we present integer formulations for the Max-CC together with preliminary results on the associated polytope. We also propose a Branch-and-Cut algorithm in order to solve instances generated from the TSPLIB.

F. Hadlock, Finding a maximum cut of a planar graph in polynomial time, SIAM Journal of Computing vol. 4 (1975) 221-225.

Haglin, Venkatesan, Approximation and Intractability results for the maximum cut problem and its variants, IEEE Trans. on computers 40 (1991) 110-113.

^{*}Intervenant

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