
On the Polytope of p-Fixed Cardinality Linear Ordering Problem

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Résumé

Linear Ordering Problem (LOP) has received significant attention in different areas of application, ranging from archeology and scheduling to economics and even mathematical psychology. It is classified as a NP-hard problem.

Assume a complete oriented graph on n vertices. A permutation of the elements of this finite set of vertices is a linear order (LO). Now let p be a fixed integer number less than or equal to n . p -Fixed Cardinality Linear Ordering Problem (p -FCLOP) is looking for a subset of vertices containing p nodes and a linear order on this subset of nodes.

Graphically, there exists exactly one oriented arc between any pair of vertices in an LOP feasible solution, which is also a cycle free digraph and the objective is to maximize the sum of the weights of all the arcs present in the solution. In p -FCLOP, there exists a subset of vertices of which are selected to be active and exactly one oriented arc between any pair of active vertices. The objective is to find the best subset of nodes that maximize the sum of the weights of all the arcs in the solution. Graphically, a feasible solution is a complete cycle-free digraph on p vertices plus a set of $n-p$ vertices not connected to any of the other vertices.

There are several studies available in the literature focused on polyhedral aspects of linear ordering problem as well as various exact methods and heuristics.

Linear ordering problem is already known as a NP-hard problem. However one sees that there exist many instances in the literature which CPLEX can solve them in less than 10 seconds for the case when $p = n$, but it runs out of memory once the cardinality number is limited to be less than n .

In this study, we have focused on Polyhedral approach and determined the dimension of the p -FCLOP polytope. It is proved in this work that the dimension of p -FCLOP is depending to the cardinality number for the three cases 1) $p = n$, 2) $p = n-1$, and 3) $p < n-2$. After determining the dimension of the polytope in these cases, we have identified several classes of facet defining inequalities and also numerically tested the impact of adding these new facet defining inequalities on the computational efficiency. Numerical results show a very significant improvement and reduction in the integrality gap.

The numerical results were carried out using CPLEX 12.4 on an "Intel Core2 Quad CPU 2.50" machine.

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Mots-Clés: Linear ordering problem, Polyhedral, Integer programming, Combinatorial optimization