Scoring rules, preferences restrictions and the Strong Borda's Paradox

Eric Kamwa[*][1] and Fabrice Valognes[2]

[1]Centre de Recherche en Economie et Management (CREM) – CNRS : UMR6211, Universite de Rennes 1, Université de Caen Basse-Normandie – 19 rue Claude Bloch 14032 Caen Cedex, France
[2]Centre de Recherche en Economie et Management (CREM) – CNRS : UMR6211, Universite de Rennes 1, Université de Caen Basse-Normandie – France

Résumé

In the voting theory, there are three well known preference restrictions that help in avoiding cycle : the Never Bottom-Ranked Restriction (NBR) (Black 1958, Saari and Valognes 1999),
the Never Middle-Ranked Restriction (NMR) (Ward 1965, Sen 1966) and the Never Top-Ranked Restriction (NTR) (Vickery 1960, Sen 1966). Nonetheless, for some voting rules, these restrictions do not prevent the Strong Borda Paradox (SgBP) i.e the election of the Condorcet loser (a candidate that loses all his pairwise comparisons). In three-candidate elections, Lepelley et al. (2000) provided a representation of the limit probability of the SgBP for all the scoring rules and they concluded that: in three-candidate elections, the Strong Borda Paradox never occurs for all the scoring rules between the Borda rule and the Antiplurality rule. Concerning the NTR and the NMR, only little thing have been said (see Lepelley et al. 2000, Gehrlein and Lepelley, 2010). In this paper, we use an analytical approach to find out all the scoring rules that never exhibit the SgBP with three candidate under the NTR and the NMR restrictions. We show that, with three candidates, the Strong Borda Paradox never occur for all the scoring rules between the Plurality rule and the Borda rule when preferences are restricted consistently with the NTR. In this respect, we compute under the IAC hypothesis, the likelihood of the Strong Borda Paradox for the Antiplurality rule for various number of voters. We also show that, in three-candidate elections when preferences are restricted in respect with the NMR, except the Borda rule, there is no other scoring rule that is non vulnerable to the Strong Borda Paradox. We Compute, given the number of voters, the likelihood under the IAC condition, of the SgBP for the Plurality rule and the Antiplurality rule.

Mots-Clés: Voting paradoxes, Borda, Condorcet, Probabilities, Impartial Anonymous Culture

---

[*]Intervenant